SHENTON COLLEGE

Year 12 Mathematics Applications
2021 Test 3 – Time Series Analysis, Compound interest Loans, investments and reducing balance loans.
Calculator Assumed

Name: ……

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Time Allowed: 50 minutes Marks: $\frac{}{55}$

Special Materials Allowed: Formula sheet, 1 A4 page notes, Classpad, calculator.
Marks may not be awarded for untidy or poorly arranged work.

Question 1. **[6 marks: 1, 1, 2, 2]**

James invests $145 000 into a savings account that earns compound interest of 2.33% per annum.

(a) Determine the total amount of the investment over six years, if the interest is compounded:

(i) annually.

(ii) weekly.

(b) Determine the total amount of interest this account will accrue in the first year when the interest is compounded daily.

(c) Determine the minimum time, to the nearest month, for this investment of $145 000 to become $169 000 when the interest is compounded monthly.

Question 2. **[7 marks: 3, 2, 2]**

The manager of a small curtain business recorded the company earnings (in \$000's) each quarter from 2018 to the third quarter of 2021. The data is shown in the table below, together with moving average calculations.

Time (t)	Year	Quarter	Company Earnings (\$000's)	4-point centered moving averages (m)	5 point moving average (g)
1	2018	1	38		
2		2	45		
3		3	20	45.125	43.6
4		4	**A**	44.75	44.6
5	2019	1	37	44.625	39.8
6		2	43	44.5	51
7		3	21	44	42.4
8		4	76	**B**	43.8
9	2020	1	35	43.875	39.2
10		2	44	43.5	49.8
11		3	20	43	41.2
12		4	74	42.375	42.4
13	2021	1	33	41.75	**C**
14		2	41		
15		3	18		

(a) Calculate the missing entries **A, B** and **C**.

(b) From the two sets of moving averages given in the table above, which is the most appropriate moving average for the manager to consider? Justify your choice.

The regression equation for the time, t, against the four-point centred moving averages, m, is $m = -0.3034t + 46.189$.

(c) Interpret the feature of the regression equation above which highlights the trend of this time series.

Question 3. **[8 marks: 3, 1, 1, 3]**

Marcia places $14 000 into an investment account with GNI bank for 5 years. Interest in this investment account is given at 2.5% p.a. compounding monthly. At the end of each month, after the interest is calculated, Marcia places an additional $95 into the investment account.

(a) Write a recurrence relation, to give the value of the investment in the account, B_n, at the end of each month, *n.*

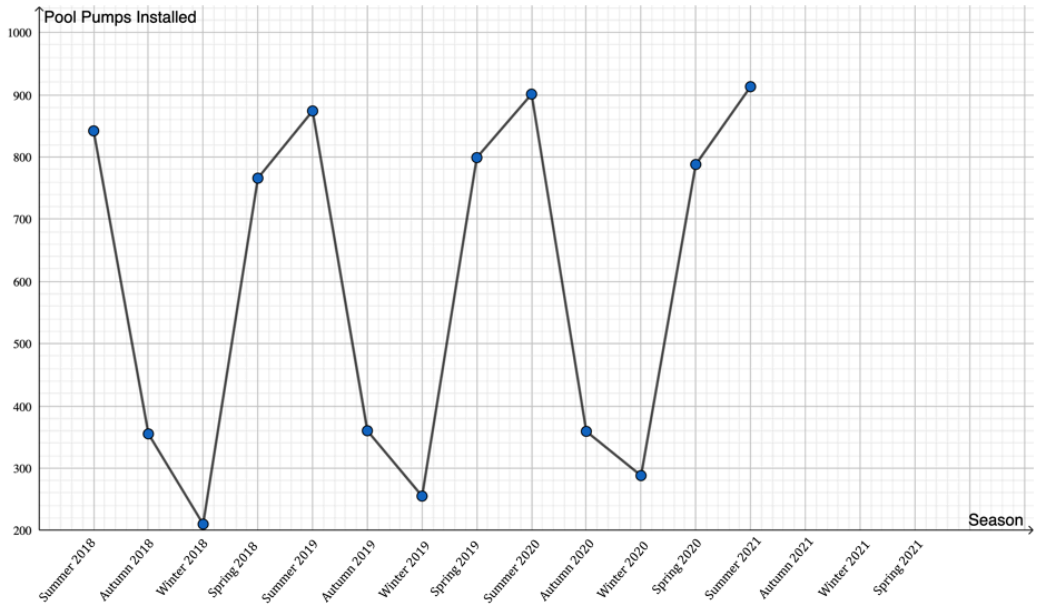
(b) Calculate how much money will be in the investment account at the end of 6 months.

(c) Calculate the final value of this investment account at the end of the 5 years.

(d) Calculate the total interest accrued by this investment account at the end of the 5 years.

Question 4. **[6 marks: 1, 2, 1, 2]**

The graph below shows the number of pool pumps installed by a local pool business each season from 2018 onwards.



The data for the next three seasons are shown in the table below.

Season	Autumn 2021	Winter 2021	Spring 2021
Pool Pumps Installed	380	310	790

(a) Complete the time series plot above by including this additional information.

(b) Determine the most appropriate moving average to smooth the data in order to predict future seasons. Justify your answer.

The seasonal indices (correct to two decimal places) are shown in the table below.

Season	Summer	Autumn	Winter	Spring
Seasonal Index	153.67%	63.32%	46.09%	

(c) Complete the table above by calculating the seasonal index for Spring.

(d) The deseasonalised number of pool pumps sold in Autumn 2019 is 562. Determine the actual number of pool pumps sold in Autumn 2019.

Question 5. **[5 marks: 2, 1, 2]**

Jacqueline recently won $40 000 from a competition in her local newspaper and wishes to invest all of her winnings into a savings account. Jacqueline has done some research and found that ZNA bank have a savings account that is 3.05% per annum compounding quarterly.

(a) If Jacqueline leaves this money in the savings account with ZNA bank for 8 years, calculate the total interest accrued from this account.

(b) Calculate the effective annual interest rate for this account.

(c) Determine the interest rate, as a percentage, that a different savings account, compounding monthly, would need to offer to achieve the same effective annual interest rate as in part (b).

Question 6. **[12 marks: 3, 2, 3, 2, 2]**

A local café has recently opened in town and is only open Monday – Friday. The total number of coffees sold each day for the first four weeks is shown in the table below.

n	Week	Day	Number of coffees sold	Weekly Mean	Number of coffees as a percentage of the weekly mean (%)
1	Week 1	Monday	458	419.6	109.15
2		Tuesday	383		**Y**
3		Wednesday	376		89.61
4		Thursday	405		96.52
5		Friday	476		113.44
6	Week 2	Monday	470	**Z**	109.56
7		Tuesday	384		89.51
8		Wednesday	379		88.34
9		Thursday	414		96.50
10		Friday	498		116.08
11	Week 3	Monday	471	435.2	108.23
12		Tuesday	388		89.15
13		Wednesday	381		87.55
14		Thursday	**X**		97.20
15		Friday	513		117.88
16	Week 4	Monday	480	443.6	108.21
17		Tuesday	399		89.95
18		Wednesday	391		88.14
19		Thursday	425		95.81
20		Friday	523		117.90

(a) Calculate the value of X, Y and Z.

(b) Describe the trend and seasonality of this data.

(c) Show how to calculate the seasonal index for Wednesday and interpret this value in the context of this situation.

The equation of the least-squares regression line for deseasonalised data is $D = 1.1966n + 420.285$ and the seasonal index for Friday is 1.1633.

(d) Use this regression line to predict the number of coffees sold on Friday of Week 6, assuming the above seasonality and trend continues.

(e) Comment on the reliability of your prediction made in part (d).

Question 7. **[11 marks: 1, 3, 2, 2, 2, 1]**

A small business took out an equipment loan of $12 000 and made monthly repayments of $750. The table below shows the progress of the loan for the first few months, with repayments and interest applied at the end of each month.

Month (n)	Balance at start of month ($)	Interest ($)	Repayment ($)	Balance at the end of the month ($)
1	12 000.00	144.00	750.00	11 394.00
2	11 394.00	136.73	750.00	10 780.73
3	10 780.73	129.37	750.00	10 160.10
4	10 160.10	**D**	750.00	**E**

(a) Show how to calculate the monthly interest of 1.2%.

(b) The recurrence relation to model the balance at the end of the month, T_n, is $T_{n+1} = aT_n - b, T_0 = c$. State the values of a, b and c.

(c) Determine the values of **D** and **E** in the table above.

(d) Determine the:

(i) number of repayments to fully pay off the loan.

(ii) amount of the final repayment.

(e) If the business opted to decrease the amount of each repayment, comment briefly on how this would change the total interest owed over the life of the loan.

END OF CLASS TEST